

**Hong Kong Mathematics Olympiad (2024/25)**

**Heats – Individual Event**

**香港数学竞赛 (2024/25)**

**初赛个人项目**

**INSTRUCTIONS**

1. Before the start of the examination, you should follow the announcement to first write your School ID, School name, seat Number and your name, in the appropriate space provided on the answer sheet.  
宣布开考前，学生须遵照司仪的指示在答题纸适当位置填写你的学校编号、学校名称、座位编号及你的姓名。
2. This paper consists of **TWO** sections, A and B.  
本试卷分**两**部分，即甲部和乙部。
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in the attached Answer Sheet. Do not write in the margins. Answers written in the margins will not be marked.  
本试卷各题均须作答，答案须写在随附的答题纸中预留的空位内。不可在边界以外位置书写。写于边界以外的答案，将不予评阅。
4. Unless otherwise stated, all answers should be given in exact numerals in their simplest form.  
除特别指明外，所有答案须以数字的真确值表达并化至最简。
5. No approximation is accepted.  
不接受近似值。
6. Unless otherwise specified, all working **NEED NOT** be shown.  
除特别指明外，**不**须列出所有算式。
7. The diagrams in this paper are not necessarily drawn to scale.  
本试卷的附图不一定依比例绘成。
8. No extra time will be given to candidates for filling in your School ID, School name, seat Number and your name after the ‘Time is up’ announcement.  
司仪宣布停笔后，考生不会获得额外时间填写你的学校编号、学校名称、座位编号及你的姓名。

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初赛个人项目

Part A

甲部

1. Let  $p = \frac{5}{1 + \frac{5}{1 + \frac{5}{1 + \dots}}}$ . Find the value of  $p$ .

设  $p = \frac{5}{1 + \frac{5}{1 + \frac{5}{1 + \dots}}}$ ，求  $p$  的值。

2. In Figure 1, a belt is used to tight 3 identical circles together such that these circles are pairwise tangent to each other. If the radius of each circle is 2 cm, find the length of the belt. (Give the answer in terms of  $\pi$ )

图一中，一条履带把 3 个大小相同的圆捆在一起，使得它们都是两两互切。若圆的半径为 2 cm，求该履带的长度。〔答案须以  $\pi$  表示〕

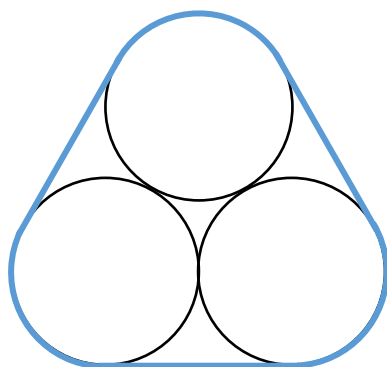


Figure 1

图一

3. Find the value of  $\frac{\tan 1^\circ \times \tan 2^\circ \times \dots \times \tan 89^\circ}{\cos^2 1^\circ + \cos^2 2^\circ + \dots + \cos^2 89^\circ}$ .

求  $\frac{\tan 1^\circ \times \tan 2^\circ \times \dots \times \tan 89^\circ}{\cos^2 1^\circ + \cos^2 2^\circ + \dots + \cos^2 89^\circ}$  的值。

4. In Figure 2, the length of the three sides of a right-angled triangle  $ABC$  are  $3x$ ,  $3x^2$  and  $2x^3$  respectively. Find the value of  $x$ .
- 图二中，直角三角形  $ABC$  的三边长度分别为  $3x$ 、 $3x^2$  及  $2x^3$ ，求  $x$  的值。

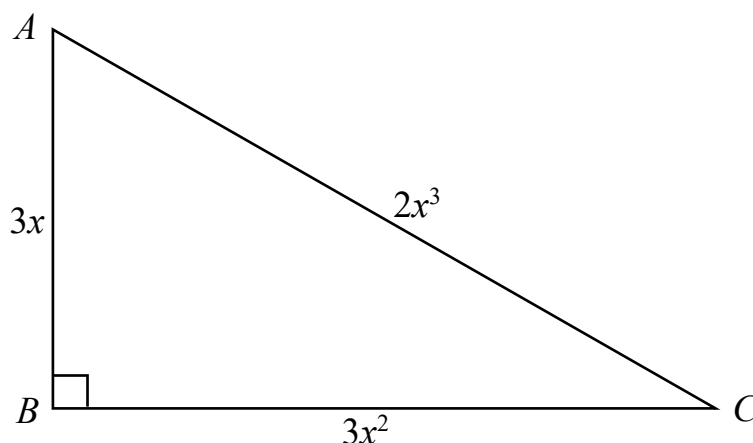


Figure 2  
图二

5. Let  $x$  be a real number. Find the minimum value of  $x^{2024} - 2024x^{1012} + y$ , where  $y$  is the largest prime factor of 2024.

设  $x$  为一实数。求  $x^{2024} - 2024x^{1012} + y$  的最小值，其中  $y$  是 2024 的最大质因子。

6. In Figure 3,  $AB$  is a diameter of the circle and  $C$  is a point on  $AB$ , and  $P$  lies on the circle such that  $PC \perp AB$ . Given that  $AC = 9$  and  $CB = 4$ , find the length of  $PC$ .

图三中， $AB$  是圆的直径， $C$  是  $AB$  上的一点，及  $P$  是圆上的一点使得  $PC \perp AB$ 。已知  $AC = 9$  及  $CB = 4$ ，求  $PC$  的长度。

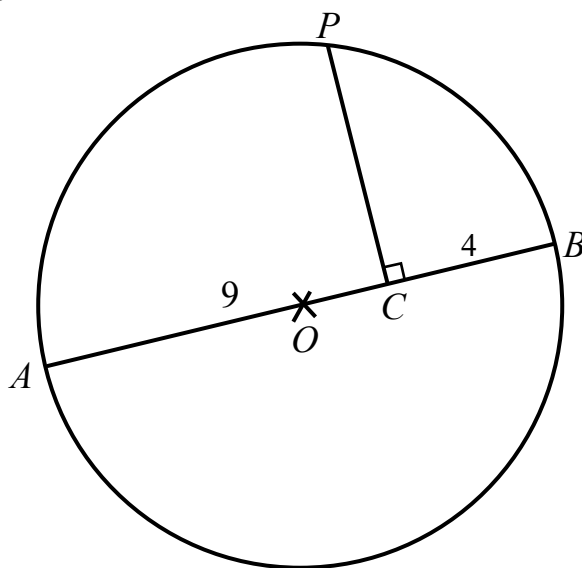


Figure 3  
图三

7. In Figure 4, two circles intersect at points  $A$  and  $B$ ,  $CD$  is an external common tangent to the circles and touch the circles at  $C$  and  $D$  respectively. A straight line passing through  $A$  intersects the circles at points  $E$  and  $F$  respectively,  $EC$  and  $FD$  intersect at point  $P$ . Given that  $\angle CBD = 60^\circ$ , find  $\angle CPD$ .

图四中，两圆相交于  $A$ 、 $B$  两点， $CD$  为一条外公切线，分别与两圆相切于点  $C$  及  $D$ 。过  $A$  任意做一条直线分别交两圆于  $E$ 、 $F$ ， $EC$  交  $FD$  于点  $P$ 。已知  $\angle CBD = 60^\circ$ ，求  $\angle CPD$ 。

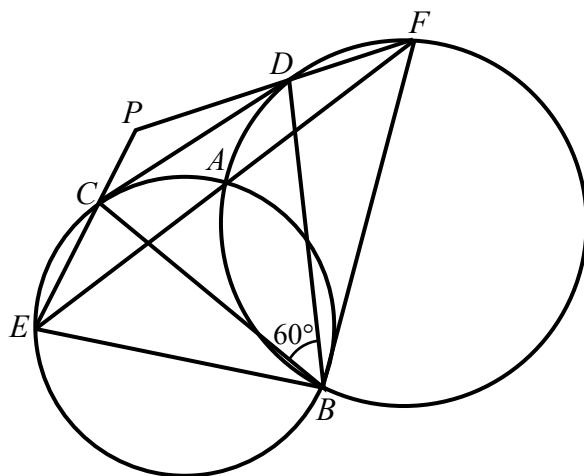


Figure 4

图四

8. Let  $x, y$  be positive integers such that  $x(2x+3)(2x+23)=5^y$ . Find the value of  $y$ .

设  $x, y$  为正整数使得  $x(2x+3)(2x+23)=5^y$ 。求  $y$  的值。

9. Given that the equation  $(x-a)(x-4)=1$  has 2 integer roots, find the value of  $a$ .

已知方程  $(x-a)(x-4)=1$  有两个整数根，求  $a$  的值。

10. Given that  $x, y$  are positive integers such that  $\frac{1}{x} - \frac{1}{y} = \frac{1}{9}$ , find the largest possible value of  $y$ .

已知  $x, y$  为正整数使得  $\frac{1}{x} - \frac{1}{y} = \frac{1}{9}$ ，求  $y$  的最大可能值。

**Part B**

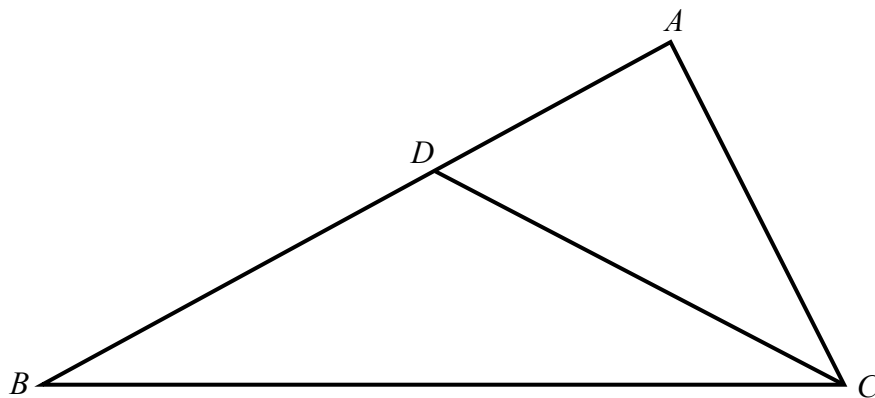
**乙部**

11. Solve the equation  $16(9^x) = 72(6^x) - 81(4^x)$ .

解方程  $16(9^x) = 72(6^x) - 81(4^x)$ 。

12. Figure 5 shows a triangle  $ABC$ ,  $\angle CAD = 2 \times \angle CBD$ ,  $CD$  is the angle bisector of  $\angle ACB$ . If  $AC = 7$ ,  $AD = 5$ , find the length of  $BC$ .

图五所示为一三角形  $ABC$ ， $\angle CAD = 2 \times \angle CBD$ ， $CD$  是  $\angle ACB$  的角平分线。若  $AC = 7$ ， $AD = 5$ ，求  $BC$  的长度。



**Figure 5**

**图五**

13. Given that one root of the equation  $x^2 - 200x + k = 0$  is 4 less than 50 times the other root of that equation, find the value of  $k$ .

已知方程  $x^2 - 200x + k = 0$  的其中一个根较另一个根的 50 倍少 4，求  $k$  的值。

14. Let  $n$  be a positive integer. Find the least value of  $n$  such that  $1^2 + 2^2 + 3^2 + \cdots + n^2$  is divisible by 2025.

设  $n$  为正整数，求  $n$  的最小值使得  $1^2 + 2^2 + 3^2 + \cdots + n^2$  可被 2025 整除。

15. Let  $y = \frac{x^2 - 2x - 3}{x^2 + 2x + 10}$  and  $x$  is a real number. Find the maximum value of  $y$ .

设  $y = \frac{x^2 - 2x - 3}{x^2 + 2x + 10}$  及  $x$  为实数。求  $y$  的最大值。

**END**

完

